TRANSVERSE OSCILLATIONS OF A CYLINDER IN UNIFORM FLOW

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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

TRANSVERSE OSCILLATIONS OF A CYLINDER
IN UNIFORM FLOW

by

Paulo Arruda Raposo

June 1976

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It has further been shown that the heuristic models previously used are inadequate to correctly predict the in-phase or the inertia component of the exciting force. Additional experiments and analysis could enable one to predict the behavior of cables subjected to ocean currents.

Transverse Oscillations of a Cylinder

in Uniform Flow

by

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NOMENCLATURE

A	amplitude of cylinder oscillation
C_d	drag coefficient as defined by equation (29)
C_m	inertia coefficient as defined by equation (30)
C_{dl}	Fourier-averaged drag coefficient
C_{ml}	Fourier-averaged inertia coefficient
D	diameter of the cylinder
F	instantaneous total force acting on the cylinder
T	period of oscillation
t	time
U	instantaneous velocity
U_m	maximum velocity in a cycle ($U_m = 2\pi A/T$)
$\bar{V}$	mean velocity of the uniform flow
A/D	dimensionless amplitude ratio
$D/\bar{V}T$	dimensionless frequency parameter
$\bar{V}T/D$	dimensionless period parameter
Re	Reynolds number ($Re = \bar{V}D/\nu$)
S	Strouhal number ($S = f_s D/\bar{V}$)
f_s	vortex-shedding frequency
f_c	cylinder oscillation frequency
ϕ	phase angle
ν	fluid kinematic viscosity
ρ_f	fluid density
ρ_s	cylinder density
ρ_r	density ratio, ρ_f/ρ_s

ACKNOWLEDGEMENT

I would like to express my great appreciation to Professor T. Sarpkaya for his guidance throughout the period of research.

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I. INTRODUCTION

Experience has shown that elastic structures of one or more degrees of freedom, such as tall slender structures, cables, etc., can extract energy from the flow about them and can develop catastrophic flow-induced oscillations. The understanding of this energy-extraction process is of paramount importance if one is either to eliminate or minimize it or to design the elastic structure such that it can withstand the oscillations under the contemplated environmental conditions.

Evidently, the real origin of the self-excited oscillations lies in the basic mechanism of the vortex-shedding process. Consequently, the prediction of such oscillations requires a tractable theoretical-formulation of the vortex-shedding phenomenon. The fact that such a formulation does not exist even for separated flow about a stationary cylinder shows the complexity of the treatment of flow about oscillating bodies. The problem would have been relatively simpler had the vortex shedding and the forces arising from it been independent of the motion of the body. Experiments show that vortex generated forces give rise to oscillations, and the oscillations modify the forces, and so on, finally resulting in a steady-state oscillation such that the Strouhal frequency of the vortex shedding nearly coincides with the natural frequency of oscillation of the body.

Mathematical models of flow-induced vibrations of bluff bodies and the response of circular cylinders to vortex shedding have been aptly described by Parkinson [1], and Currie, et al. [2] and will not be repeated here.

It appears that among the various models considered so far, the "wake-oscillator" model of Hartlen and Currie [3] attracted considerable attention among the students of vibration analysis. This model predicts some of the observed features of the self-excited oscillations even though the reasons as to why an oscillator of the van der Pol type should at all describe the self-excited oscillations of an elastic body are not yet clear. There are several coefficients in the model which must be determined experimentally. There are, at present, wide variations in the methods of measurement of some of the coefficients. For example, the damping of the vibrations of a given beam or cylinder is determined in still air or water depending on whether the self-excited oscillations take place in air or water. The added mass is assumed to be equal to the displaced mass of the cylinder. There is no distinction between the material damping as would be determined in vacuum, and the damping caused by the fluid.

It became clear, during the course of the present investigation, that the forces acting on a cylinder oscillating harmonically transverse to a uniform flow should be measured and the in-phase and out-of-phase components of the force should be determined. This phase of the investigation has been carried by Meyers [4] and Fortik [5] under the direction of Professor T. Sarpkaya, and the drag and inertia coefficients for the lift force have been determined through the use of Morison's [6] equation.

The purpose of the work reported herein is to make use of the measured force by combining it with the equation of motion of an elastically mounted, linearly damped, circular cylinder and to attempt

to predict the self-excited oscillations of the cylinder for given and directly measurable physical parameters such as the mass, length, diameter, material damping, and the natural frequency of the body, and the density and velocity of the fluid.

It will be assumed that the measurements and the predictions are in the same Reynolds number range and that the force coefficients do not measurably depend on the Reynolds number in the range under consideration.

II. ANALYSIS

A. FORCING FUNCTION

The transverse force acting on a circular cylinder undergoing harmonic oscillations in the transverse direction to a uniform flow is expressed in terms of the Morison's equation as [4]

$$C_L = \frac{F}{\frac{1}{2}\rho_f L D \bar{V}^2} = C_{ml}\ \pi^2 \left(\frac{U_m T}{D}\right) \left(\frac{D}{\bar{V}T}\right)^2 \sin\frac{2\pi}{T}t$$

$$- C_{dl} \left(\frac{U_m T}{D}\right)^2 \left(\frac{D}{\bar{V}T}\right)^2 \left|\cos\frac{2\pi}{T}t\right| \cos\frac{2\pi}{T}t \qquad (1)$$

where C_{ml} and C_{dl} are the inertia and drag coefficients, respectively. Evidently, the normalized force as well as the drag and inertia coefficients depend on the parameters $U_m T/D = 2\pi A/D$, and $D/\bar{V}T$. The first parameter is directly proportional to the relative amplitude, and the second parameter may be regarded as the frequency parameter ($D/\bar{V}T$) or the reduced velocity ($\bar{V}T/D$).

For the purpose under consideration it is more advantageous to expand the $|\cos \omega t|\cos \omega t$ term in series and to retain only the first term. This procedure yields,

$$C_L = C_{ml}\ \pi^2 \left(\frac{U_m T}{D}\right)\left(\frac{D}{\bar{V}T}\right)^2 \sin \omega t - \frac{8}{3\pi} C_{dl} \left(\frac{U_m T}{D}\right)^2 \left(\frac{D}{\bar{V}T}\right)^2 \cos \omega t \qquad (2)$$

Equation (2) could be simplified further by combining it with the equation of motion for an elastically mounted cylinder.

B. ELASTICALLY MOUNTED CYLINDER

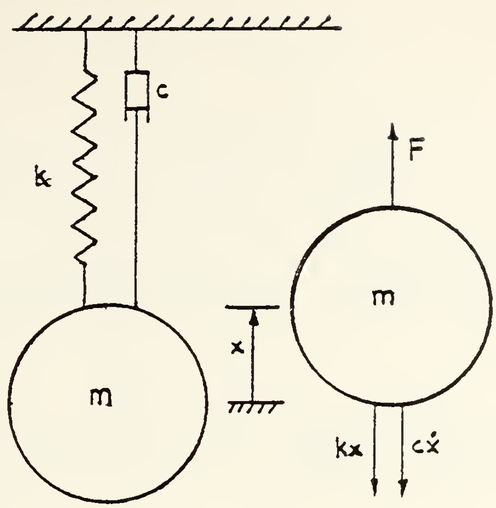

Fig. 1. Elastically mounted, linearly damped cylinder.

The equation of motion for the elastically mounted, linearly damped, and periodically forced cylinder may be written as (see Fig. 1)

$$m\ddot{x} + c\dot{x} + kx = F = \frac{1}{2}\rho_f \, DL \, \bar{V}^2 \, C_L \tag{3}$$

where m represents the mass of the cylinder, c the linear damping, and k the spring constant. The derivatives of x are taken with respect to real time t, i.e., $\frac{d^2x}{dt^2} = \ddot{x}$, etc.

Defining,

$$\omega_m = \sqrt{k/m}\,, \quad \omega_n = 2\pi f_n\,, \quad \tau = \omega_n t\,, \quad x/D = x_r\,,$$

$$\zeta = \frac{c}{2m\,\omega_n}\,, \quad \omega_o = f_s/f_n = S\bar{V}/f_n D\,, \quad X_r = A/D$$

$$\Omega = f_c/f_n\,, \quad a = \frac{\rho f \; LD^2 \; f_s^2}{2m\,\omega_n^2\,S^2\omega_o^2}\,, \qquad \ddot{x}_r = \frac{\partial^2 x_r}{\partial\tau^2}$$

equation (3) may be reduced to

$$\ddot{x}_r + 2\zeta\dot{x}_r + x_r = a\omega_o^2[C_{ml}\ \pi^2(2\pi X_r)(\frac{\Omega S}{\omega_o})^2 \sin \Omega\tau$$

$$- \frac{8}{3\pi} C_{dl}(2\pi X_r)^2 (\frac{\Omega S}{\omega_o})^2 \cos \Omega\tau] \qquad (4)$$

Defining $\rho_r = \rho_f/\rho_s$ and noting that $\rho_r = 2\pi^3 aS^2$, equation (4) reduces to

$$\ddot{x}_r + 2\zeta\dot{x}_r + x_r = \rho_r X_r \Omega^2(C_{ml} \sin \Omega\tau - \frac{16}{3\pi^2} X_r C_{dl} \cos \Omega\tau) \qquad (5)$$

or by a change of variable from x_r to $y_r = x/A$, and further simplification one has

$$\ddot{y}_r + 2\zeta\dot{y}_r + y_r = \rho_r \Omega^2(C_{ml} \sin \Omega\tau - \frac{16}{3\pi^2} X_r C_{dl} \cos \Omega\tau) \qquad (6)$$

Equation (6) may be written as

$$\ddot{y}_r + 2\zeta\dot{y}_r + y_r = E\Omega^2 \sin (\Omega\tau - \alpha) \qquad (7)$$

in which

$$E = \rho_r \sqrt{C_{ml}{}^2 + [(\frac{16}{3\pi^2})(\frac{A}{D})\ C_{dl}]^2} \qquad (8)$$

and

$$\alpha = \arc\tan \frac{16A\ C_{dl}}{3\pi^2\ D\ C_{ml}} \qquad (9)$$

The coefficient $E\Omega^2$ may also be expressed in terms of the maximum lift coefficient defined by

$$F = \frac{1}{2} \rho_f\ \bar{V}^2 DL\ (C_L)_m \sin (\Omega\tau - \alpha) \qquad (10)$$

Inserting equation (10) into equation (3) and simplifying one has

$$E\Omega^2 = \frac{\frac{1}{2}\rho_f \bar{V}^2 DL(C_L)_m}{m\,A\,\omega_n^2} \tag{11}$$

or

$$E = \rho_r \frac{(\frac{\bar{V}T}{D})^2}{2\pi^3} \frac{(C_L)_m}{(A/D)} \tag{12}$$

Combining equations (12) and (8) and simplifying one has

$$\frac{(C_L)_m}{(A/D)} = \frac{2\pi^3}{(\frac{\bar{V}T}{D})^2} \sqrt{C_{ml}^{\,2} + (\frac{16}{3\pi^2}\,\frac{A}{D}\,C_{dl})^2} \tag{13}$$

It should be noted that if (A/D) C_{dl} were independent of A/D, then it is easy to show that $(C_L)_m/(A/D)$ would also be independent of A/D since the experiments have shown that C_{ml} is independent of A/D. This would make the results independent of A/D, a conclusion which is contrary to observations. This proves that (A/D) C_{dl} and $(C_L)_m/(A/D)$ are indeed dependent on A/D in a complex manner. But this dependence is rather weak. That is why slight changes in, for instance, ρ_r can lead to large changes in ζ, or Ω.

C. SOLUTION OF THE EQUATION OF MOTION

As shown above, a convenient form of the equation of motion is

$$\ddot{y}_r + 2\zeta\dot{y}_r + y_r = E\Omega^2 \sin(\Omega\tau - \alpha) \tag{14}$$

1. Transient Solution

The Homogeneous equation corresponding to equation (14) is

$$\ddot{y}_r + 2\zeta\dot{y}_r + y_r = 0 \tag{15}$$

Its characteristic equation reduces to

$$s^2 + 2\zeta s + 1 = 0 \tag{16}$$

whose roots are given by

$$s_{1,2} = -\zeta \pm i\sqrt{1 - \zeta^2} \tag{17}$$

Considering only the underdamped case, it follows that $\zeta < 1$. Therefore, according to Boyce and Di Prima [7], the solution to the corresponding homogeneous equation, that is, the transient solution is given by

$$(y_r)_h = Y_h e^{-\zeta\tau} \sin(\sqrt{1 - \zeta^2}\ \tau - \beta) \tag{18}$$

where Y_h and β will be determined from the initial conditions.

2. Steady-State Solution

The particular solution to equation (14) is a steady-state oscillation of the same frequency of the excitation. Thus, as indicated in Thomson [8], the particular solution is assumed to be of the form

$$(y_r)_p = Y_p \sin[(\Omega\tau - \alpha) - \phi] \tag{19}$$

where ϕ is the phase of the displacement with respect to the excitation, as shown in Fig. 2, and is given by

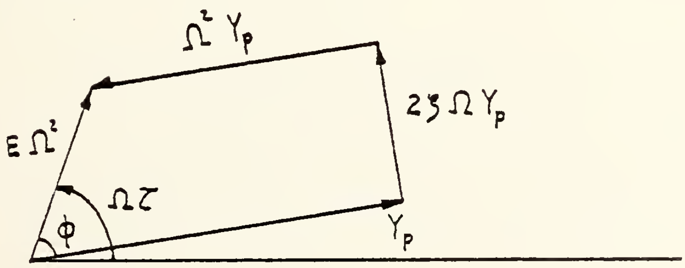

Fig. 2. Vector diagram.

and

$$\phi = \arc \tan \frac{2\zeta\Omega}{1-\Omega^2} \tag{20}$$

and Y_p is given by

$$Y_p = \frac{E\Omega^2}{\pm \sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \tag{21}$$

Adding the transient and the steady-state solutions, the following complete solution is obtained

$$y_r = Y_h \, e^{-\zeta\tau} \sin\left(\sqrt{1-\zeta^2}\,\tau+\beta\right) + Y_p \sin(\Omega\tau-\alpha-\phi) \tag{22}$$

Also, its derivative with respect to τ is given by

$$\dot{y}_r = Y_h\{-\zeta e^{-\zeta\tau} \sin\left(\sqrt{1-\zeta^2}\,\tau+\beta\right) + e^{-\zeta\tau} \sqrt{1-\zeta^2}\,\tau+\beta)\}$$

$$+ Y_p \, \Omega \cos(\Omega\tau-\alpha-\phi) \tag{23}$$

Applying the first initial condition that $\tau = 0$ corresponds to $y_r = 0$, one finds

$$Y_h = - \frac{Y_p \sin(-\alpha-\phi)}{\sin \beta} \tag{24}$$

Applying the second initial condition that $\tau = 0$ corresponds to $\dot{y}_r = 0$, one obtains

$$0 = Y_h(-\zeta \sin\beta + \sqrt{1-\zeta^2} \cos\beta) + Y_p\Omega \cos(-\alpha-\phi) \tag{25}$$

Inserting the expression for Y_h given by equation (24) in equation (25),

one has

$$\beta = \text{arc tan } \left(\frac{\sqrt{1-\zeta^2}}{\zeta+\Omega \cot(-\alpha-\phi)}\right) \tag{26}$$

Also, recalling that as $\tau \to \infty$, $x \to -A \sin \beta$, and that $y_r = x/A \to -\sin \theta$, it is seen from equation (22) that $Y_p = -1$. Introducing this value in equation (21) and noting that $E \geq 0$, one obtains

$$E = \frac{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}}{\Omega^2} \tag{27}$$

Substituting the expressions for Y_p and Y_h into equation (22), one obtains the following complete solution of the equation of motion

$$y_r = \frac{\sin(-\alpha-\phi)}{\sin \beta} e^{-\zeta\tau} \sin (\sqrt{1-\zeta^2}\, \tau+\beta) - \sin (\Omega\tau-\alpha-\phi) \tag{28}$$

The results obtained through the use of equation (28) will be presented in connection with the discussion of results.

III. DISCUSSION OF RESULTS

The time dependent force acting on the cylinder was expressed, as indicated previously, in terms of C_{ml} and C_{dl}. Assuming this force to be applied to an elastically mounted and linearly damped cylinder, the equation of motion for the cylinder was developed and solved in terms of the given parameters and experimentally obtained coefficients. Two of these coefficients, C_{ml} and C_{dl}, inertia and drag coefficients respectively, were introduced in the expression of the time dependent force acting on the cylinder. Using the experimental apparatus and procedure described in [4], data were obtained, which together with the results from [4], were used to plot C_{ml} and C_{dl} as a function of $\bar{V}T/D$ for A/D = 0.25, 0.50, 0.75 and 0.84, as shown in Figs. 3 and 4.

The results plotted in Fig. 3 show that for the region of interest, i.e., for $\bar{V}T/D$ in the range of 4 to 7, and for the precision of the experiments, C_{ml} is independent of A/D.

On the other hand, C_{dl} is dependent on both $\bar{V}T/D$ and A/D as shown in Fig. 4. As noted previously, (A/D) C_{dl} and $(C_L)_m/(A/D)$ are not constants. Furthermore, it can be shown that $(C_L)_m/(A/D)$ decreases as A/D increases. Figures 5, 6, 7 and 8 show $(C_L)_m$ as a function of $\bar{V}T/D$ for A/D = 0.25, 0.50, 0.75 and 0.84 respectively.

Therefore, an iterative procedure is needed to determine the complete solution of the equation of motion for a specific case, as indicated below.

Firstly, for a given $\bar{V}T/D$, C_{ml} is obtained from Fig. 3. Then, for a specific relative amplitude A/D and the same $\bar{V}T/D$, C_{dl} is obtained from

Fig. 4, and the value of $(C_L)_m$ is obtained from Figs. 5, 6, 7 or 8, or alternatively from equation (13). Then, E is calculated using equation (12), since ρ_r is known. Substituting this value for E in equation (27), a value for ζ is calculated, since $\Omega = (\frac{\bar{V}T_n}{D})/(\frac{\bar{V}T}{D})$. A plot of 1/E versus Ω for specific values of ζ is shown in Fig. 9. Then, the values for α, ϕ and β are calculated from equations (9), (20) and (26), respectively. Substituting these results into the equation of motion and plotting the results, one obtains curves similar to those shown in Figs. 10, 11 and 12. These figures show clearly the effect of ζ on the response of the system. A lowly damped system results in an initial overshoot of the amplitude before it reaches a steady-state. A highly damped system, on the other hand, reaches its steady-state gradually from its initial rest position.

Evidently, one can perform various parametric studies on the effect of damping, density ratio, etc., on the transient and steady-states of the oscillation provided that sufficient experimental data are available to obtain accurately enough the drag and inertia coefficients. It is also evident that the practical problem is the determination of whether a cylinder of given natural frequency, density ratio, internal damping, etc., will be self excited in a fluid of given velocity.

The determination of this problem requires an iteration not only over $\bar{V}T/D$ but also over A/D in order to obtain the appropriate values of the drag and inertia coefficients. Preliminary attempts to resolve this aspect of the problem have resulted in the computer program given in Appendix A. This program needs to be improved with additional data particularly to the lower and higher values of A/D where the force transfer coefficients vary most rapidly with A/D.

Then, it will be possible to improve the program to predict with greater precision the range of self-excited oscillations as well as their amplitude.

A computer program for the complete solution given by equation (28) and for the plotting of the results is given in Appendix B.

It is evident from the foregoing discussion that the prediction of the characteristics of the self-excited oscillations is based on the measured time-dependent transverse force rather than on an assumed variation of the lift coefficient through the use of a heuristic model [3]. It was, therefore, desirable to compare the components of the alternating force obtained from the experimental data with those predicted by the Hartlen and Currie [3] model. For this purpose the in-phase and out-of-phase components of the Hartlen and Currie transverse force coefficients have been written equal to the corresponding components of the model used herein [see equation (2)]. This procedure yielded

$$C_d(\text{Hartlen and Currie}) = -\frac{32\pi\,(\frac{A}{D})^2}{3(\frac{\bar{V}T}{D})^2}\,C_{dl} \tag{29}$$

and

$$C_m(\text{Hartlen and Currie}) = \frac{2\pi^3\,(\frac{A}{D})}{(\frac{\bar{V}T}{D})^2}\,C_{ml} \tag{30}$$

in which C_{ml} and C_{dl} are the drag and inertia coefficients evaluated in the present investigation.

Equations (29) and (30) have been evaluated for A/D = 0.25 and 0.50 and the results are plotted as a function of f_c/f_s in Figs. 13 and 14.

A similar plot obtained from [3] is shown in Fig. 15 for $A/D = 0.2$. Even though the A/D values are not exactly identical, Figs. 13, 14 and 15 could certainly be compared as far as the overall variation of C_m and C_d are concerned. It is evident that C_d curves are fairly similar and that the Hartlen and Currie model could be adjusted sufficiently to yield similar C_d values for identical A/D values. However, C_m values which are comparable only for $f_c/f_s < 1$ show opposite variations. It has been previously stated that the use of an added mass coefficient in terms of the displaced mass of the body is not sufficient and that the inertia coefficient of a body oscillating in the transverse direction to a uniform flow could be significantly different from its displaced mass. It suffices to conclude that the Hartlen and Currie model does not predict C_m in accordance with the observations and that the overall predictions of the model may be attributed partly to the adjustment of the other free parameters such as the damping ratio, the density ratio, etc.

IV. CONCLUSIONS

The results presented herein have shown that (a) a cylinder can undergo self-excited oscillations for certain values of the ratio of the natural frequency of the cylinder to the Strouhal frequency of the vortices (f_n/f_s), internal damping of the cylinder (ζ), and the density ratio (ρ_r); (b) the drag and inertia coefficients must be determined experimentally as a function of A/D and $\bar{V}T/D$ and must be incorporated into the equation of motion; (c) the prediction of the self-excited oscillations and their amplitude will be possible with the acquisition of additional data for the transverse force; and that (d) the results presented herein are quite encouraging and certainly a reflection of the combination of the fluid dynamical and vibrational aspects of the phenomenon unlike the heuristic models previously proposed.

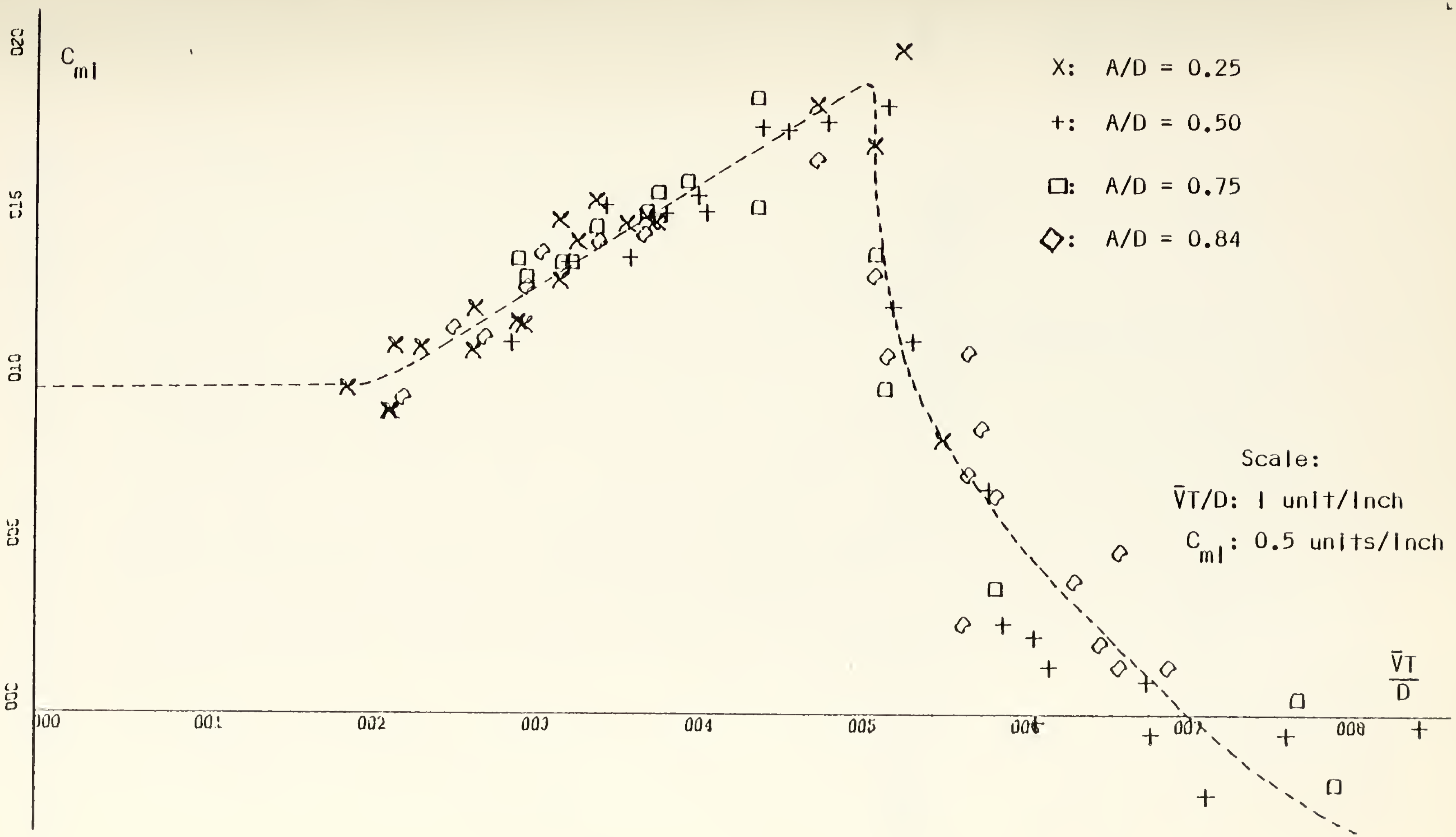

Fig. 3. Inertia coefficient versus $\bar{V}T/D$ for various values of A/D.

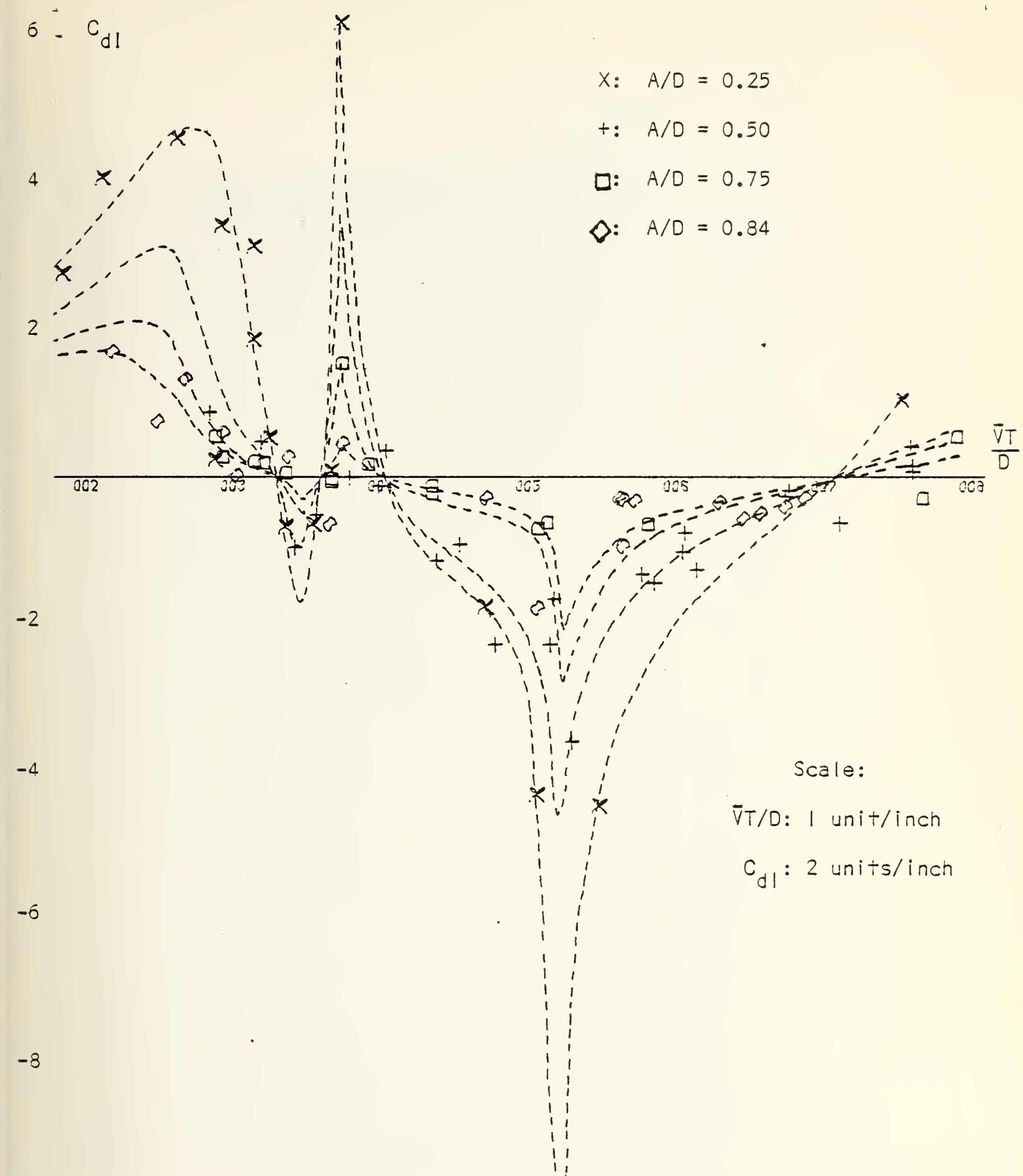

Fig. 4. Drag coefficient versus $\bar{V}T/D$ for various values of A/D.

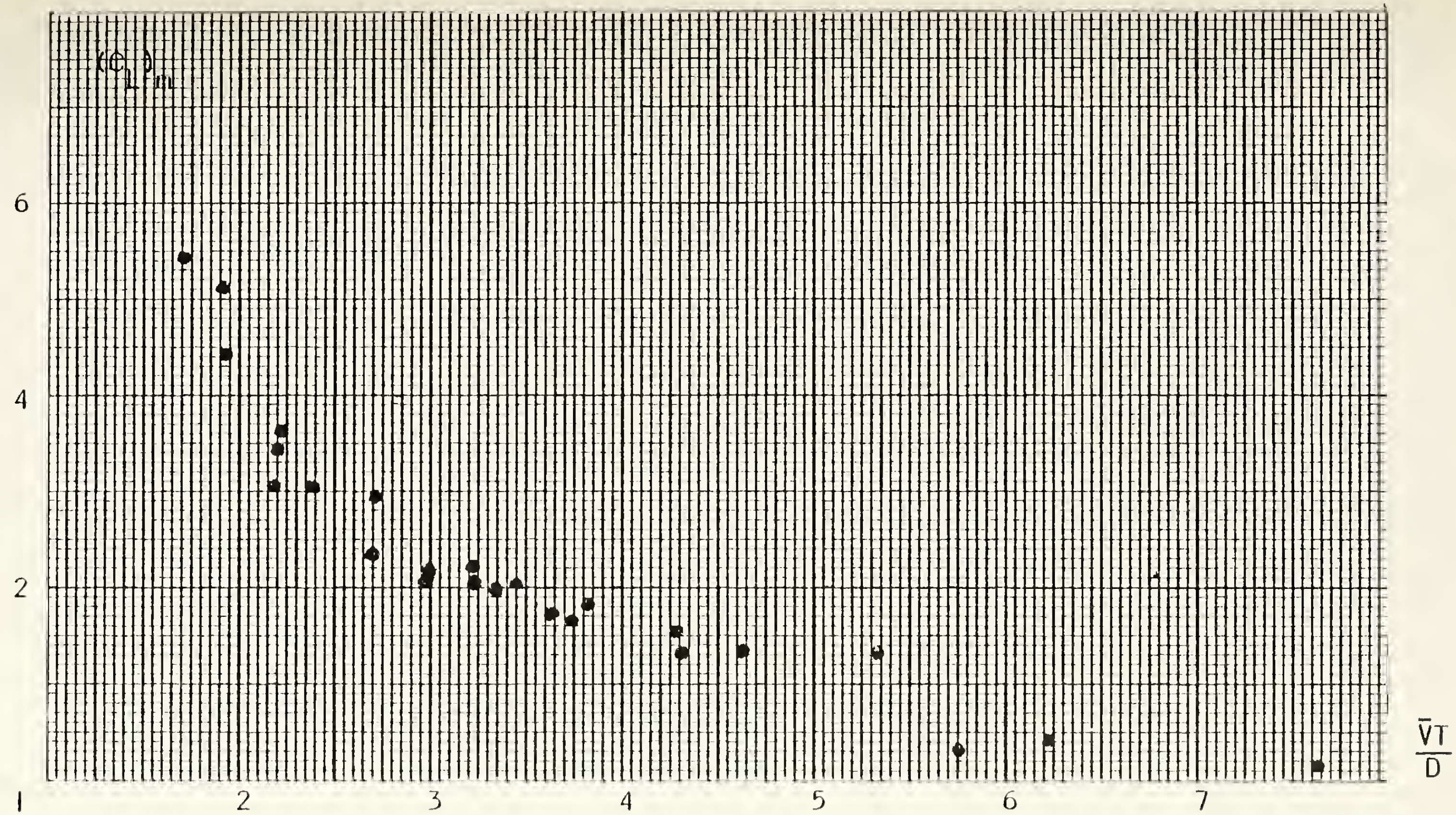

Fig. 5. Normalized amplitude of the lift force versus $\bar{V}T/D$ for A/D = 0.25.

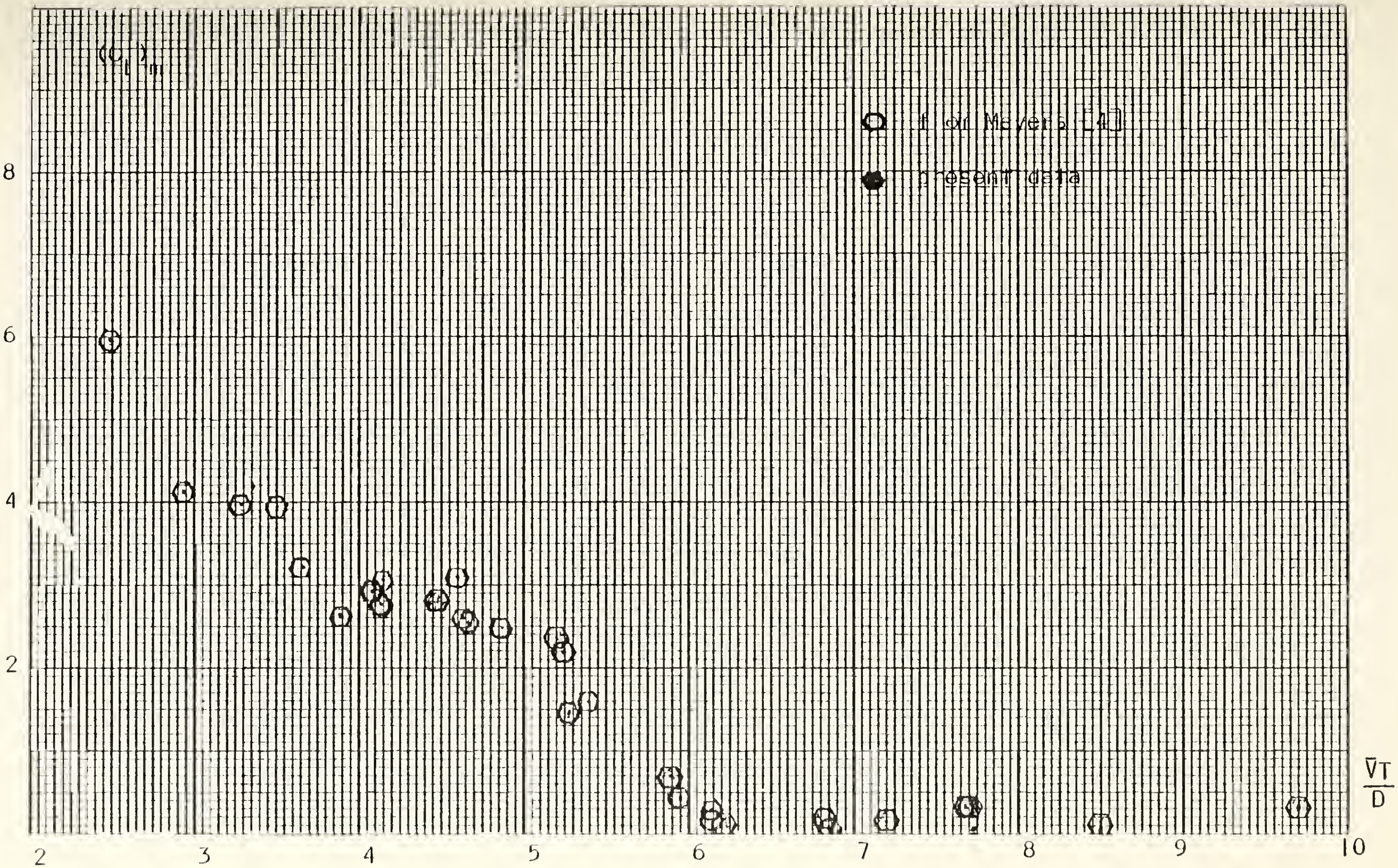

Fig. 6. Normalized amplitude of the lift force versus $\bar{V}T/D$ for $A/D = 0.50$.

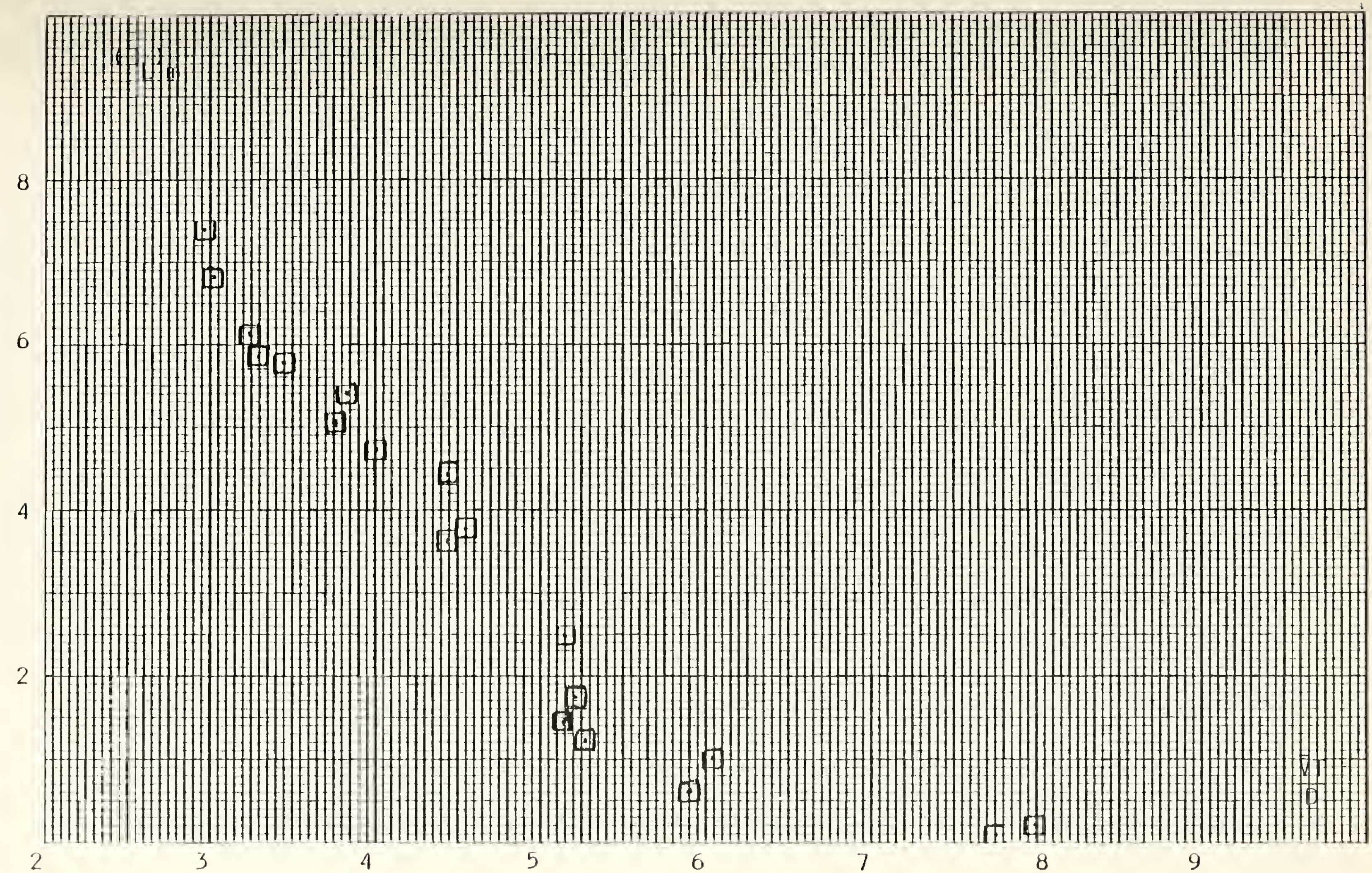

Fig. 7. Normalized amplitude of the lift force versus $\bar{V}T/D$ for $A/D = 0.75$.

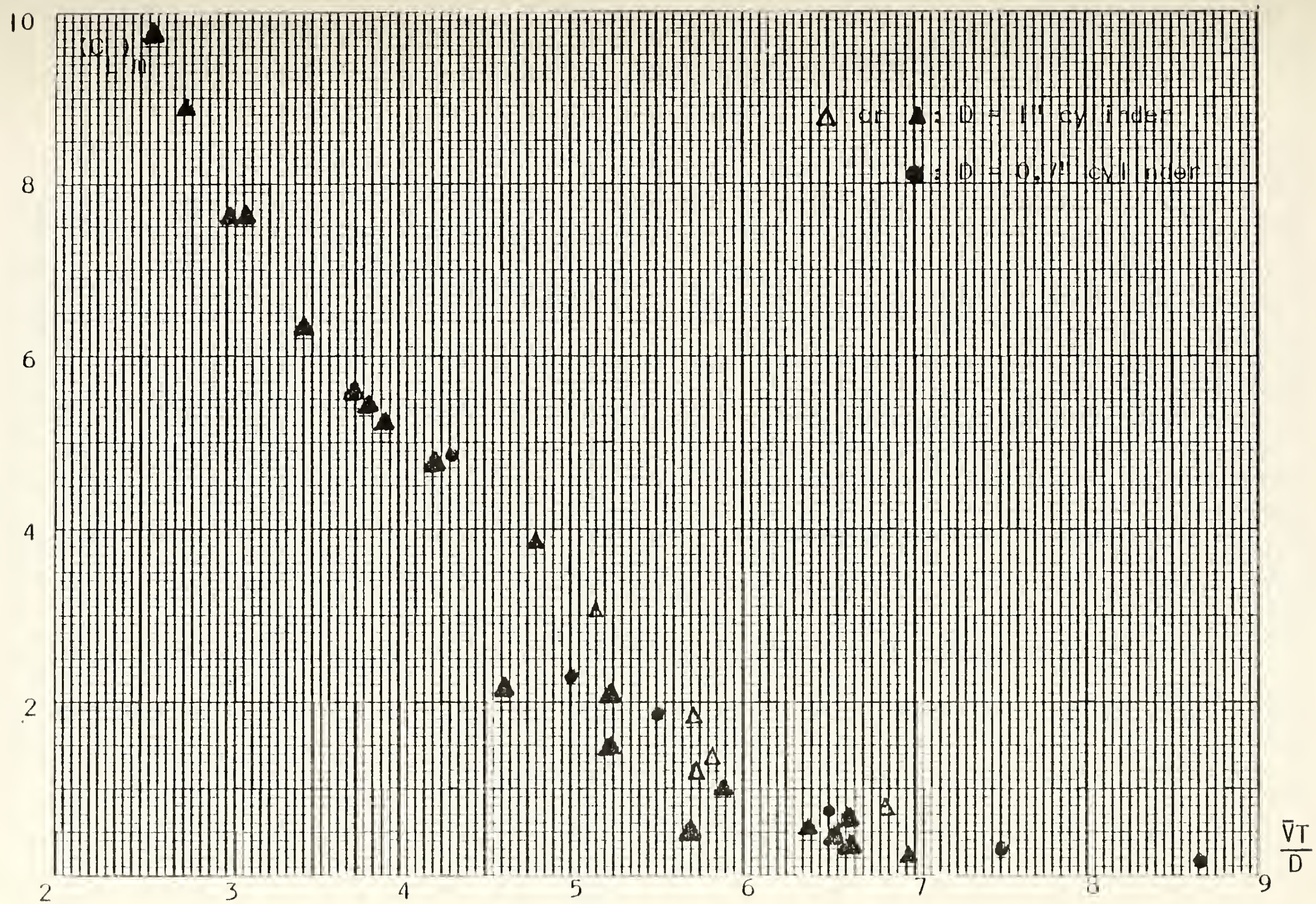

Fig. 8. Normalized amplitude of the lift force versus $\bar{V}T/D$ for A/D = 0.84.

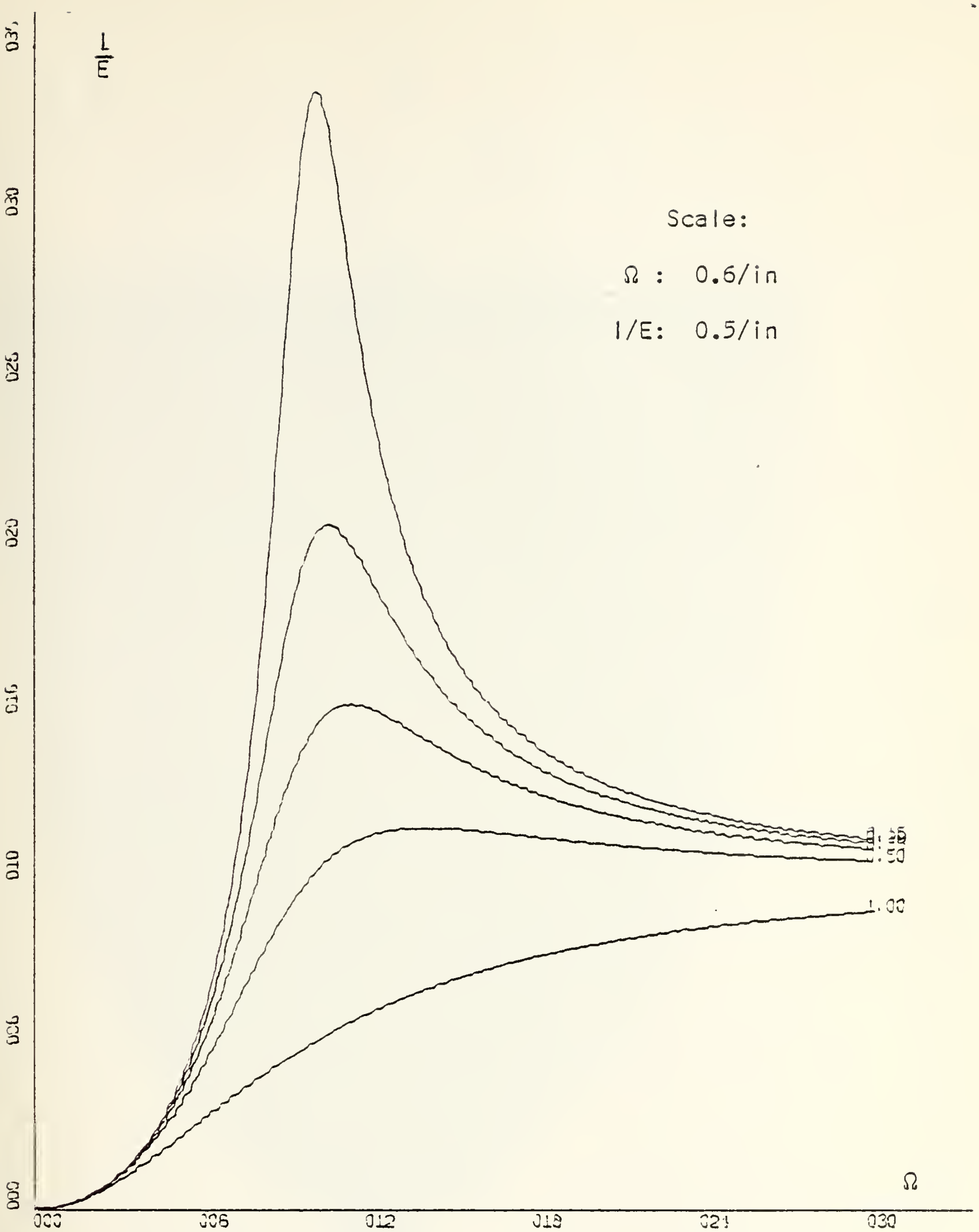

Fig. 9. The function I/E versus Ω for various values of the damping ratio ζ.

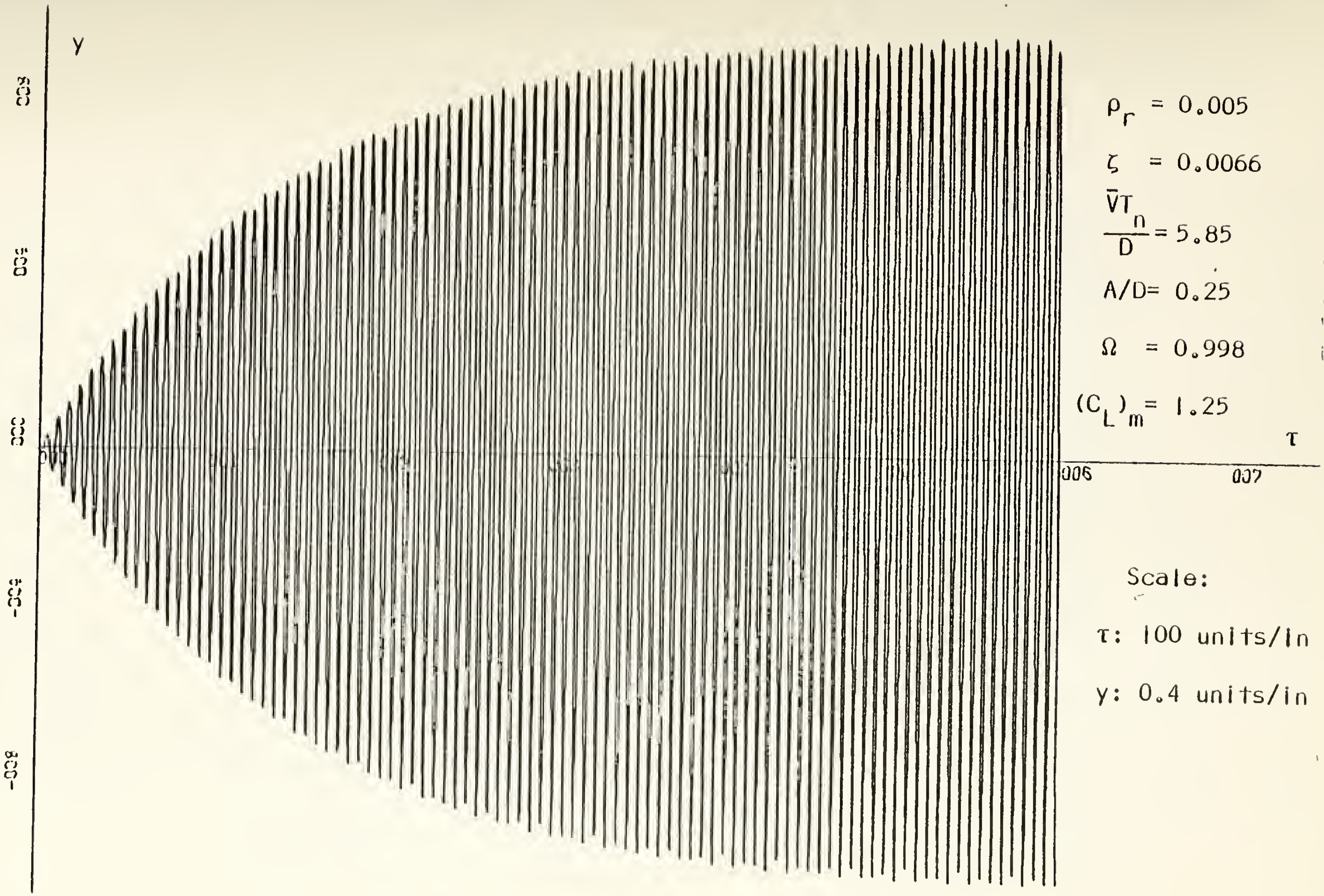

Fig. 10. Sample calculations for the transient and steady-state oscillations of a cylinder for the specific parameters indicated.

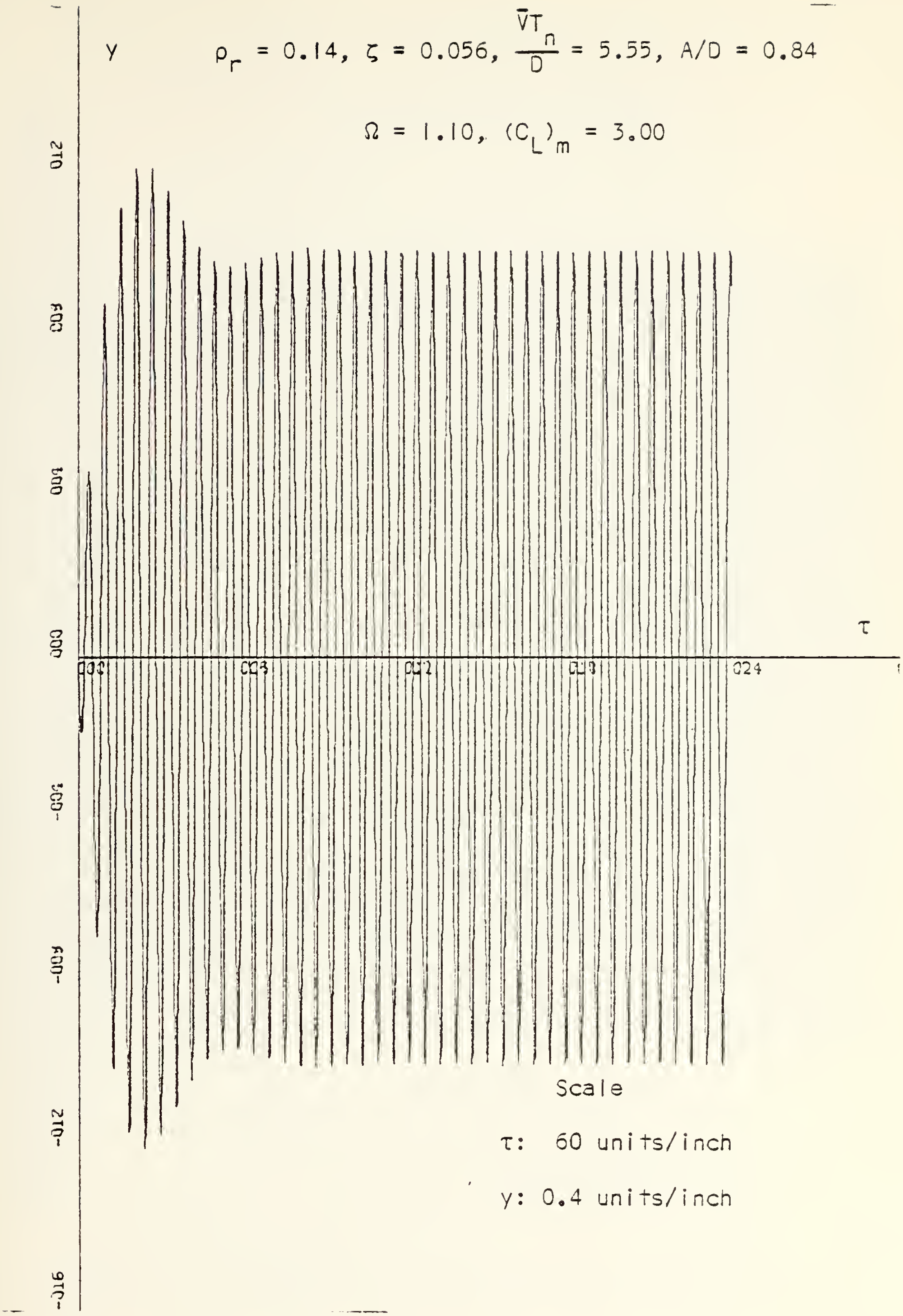

Fig. 11. Sample calculations for the transient and steady-state oscillations of a cylinder for the specific parameters indicated.

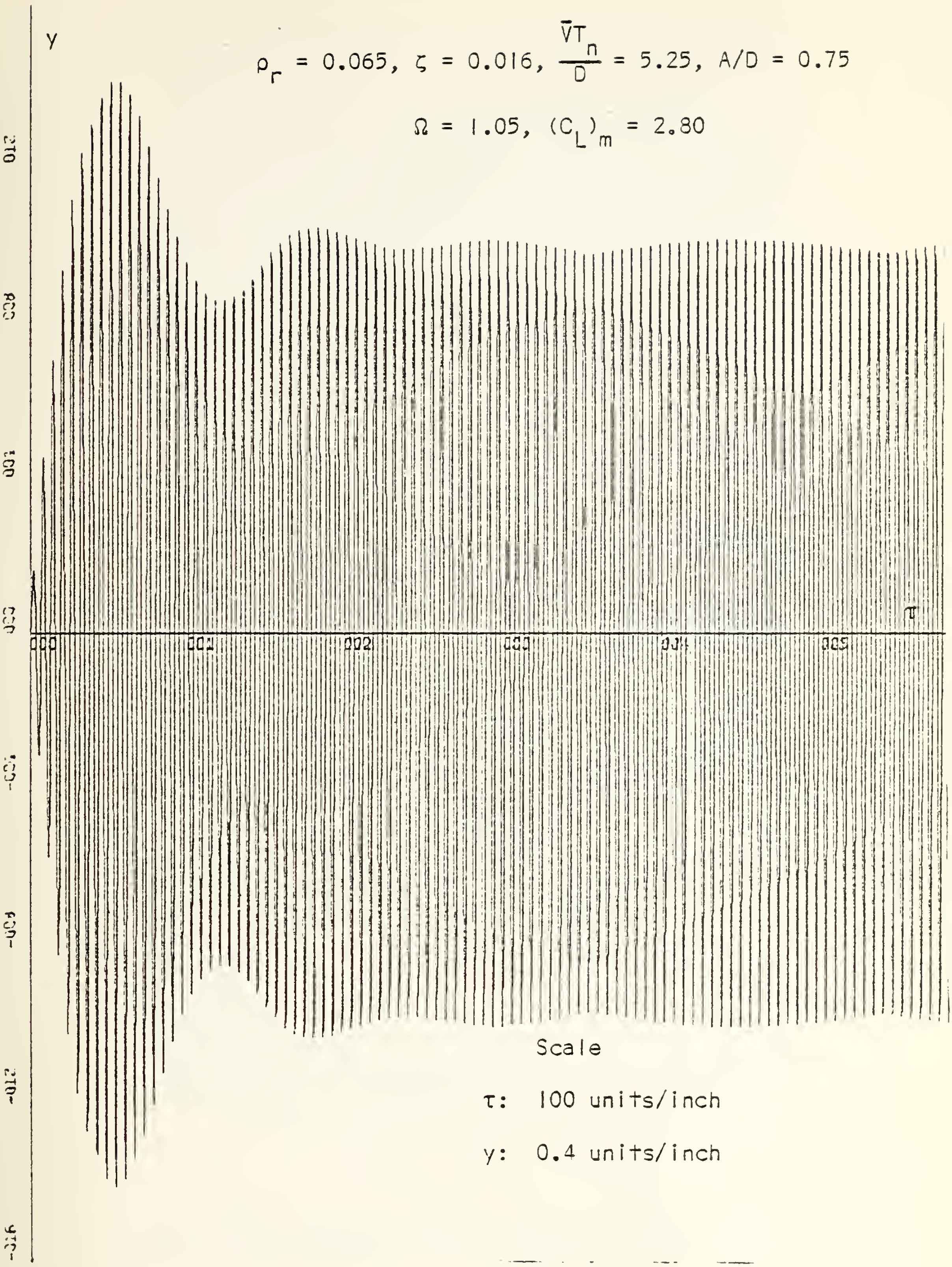

Fig. 12. Sample calculations for the transient and steady-state oscillations of a cylinder for the specific parameters indicated.

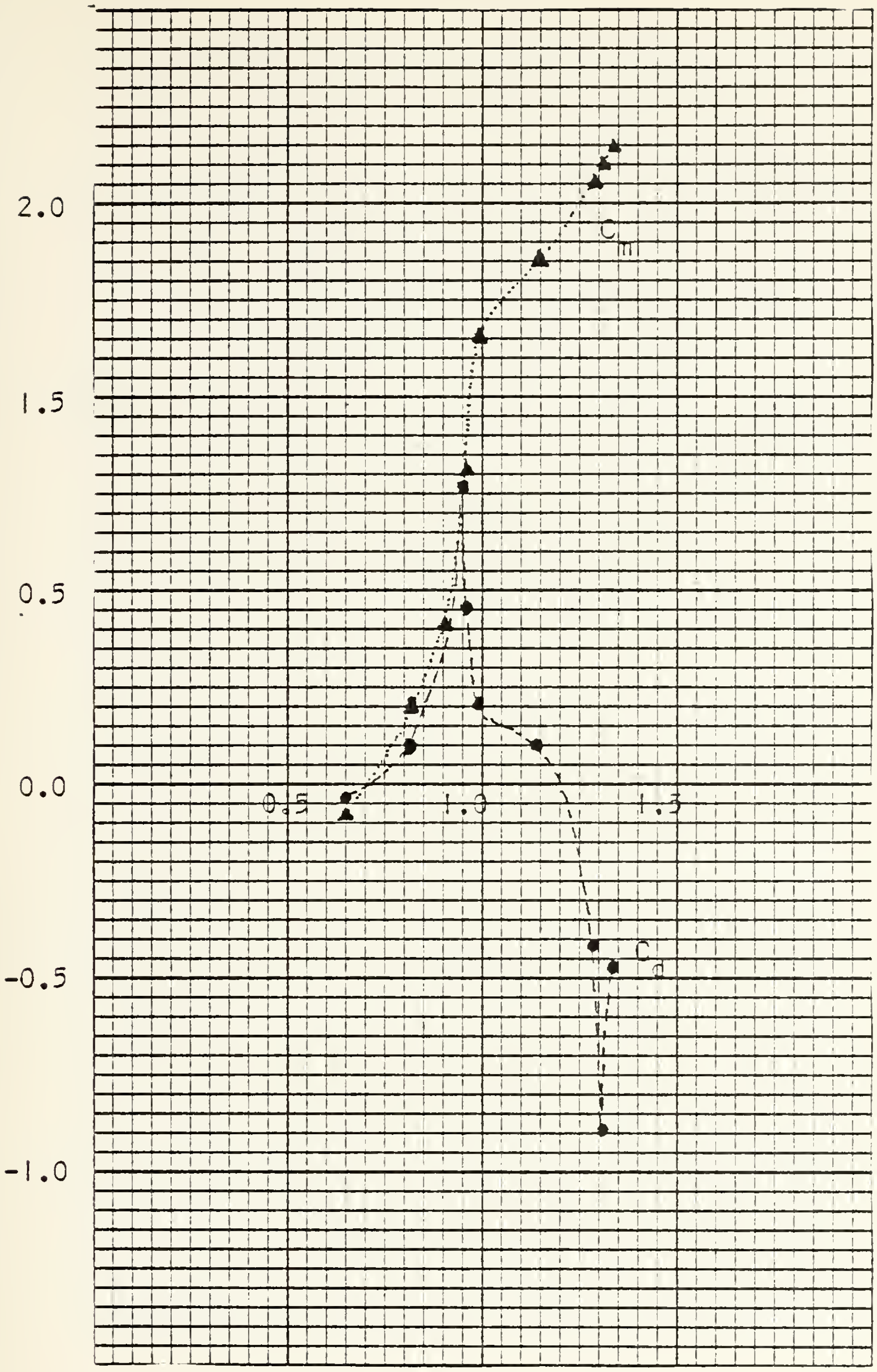

Fig. 13. C_d and C_m versus f_c/f_s for A/D = 0.25.

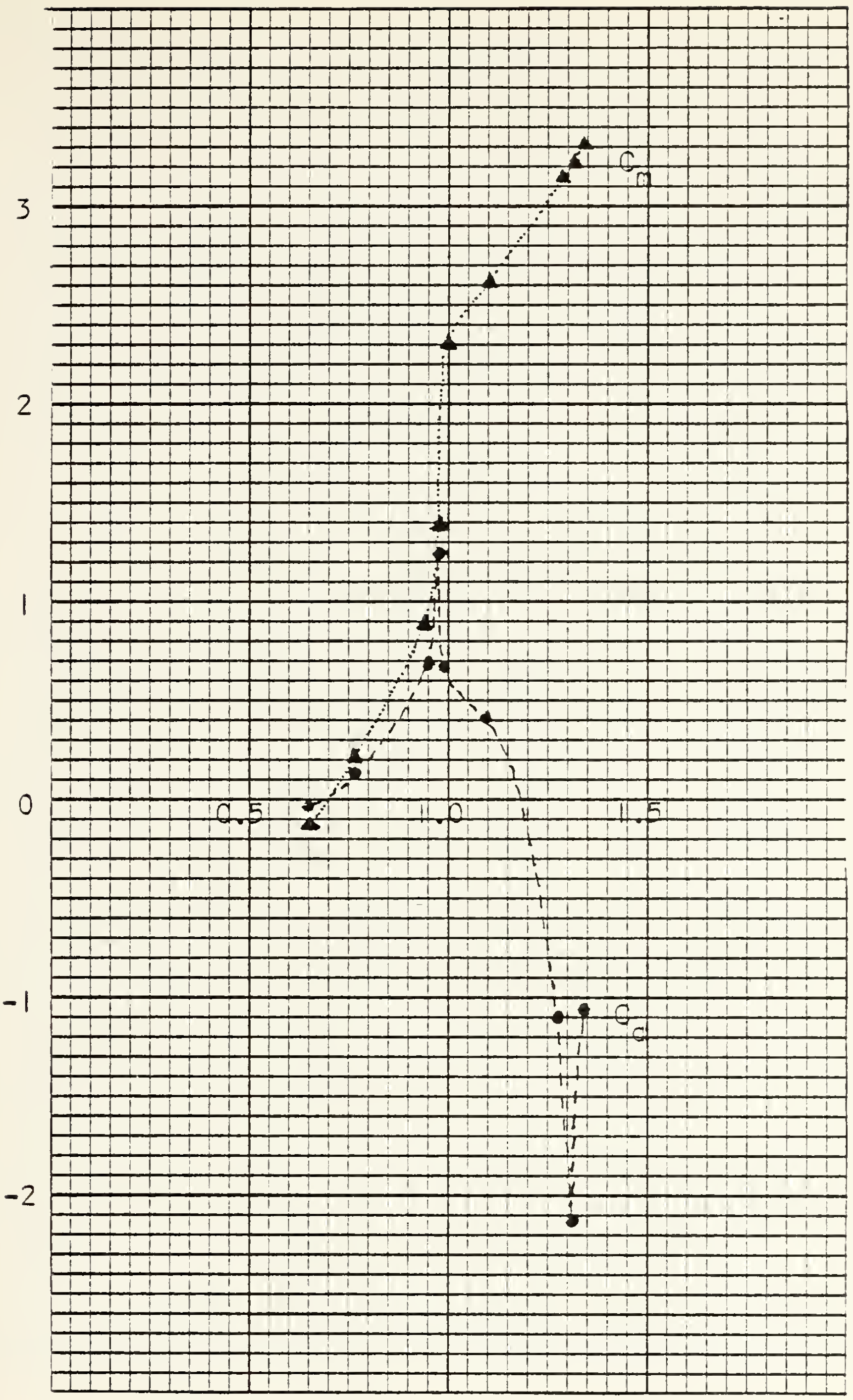

Fig. 14. C_d and C_m versus f_c/f_s for A/D = 0.50.

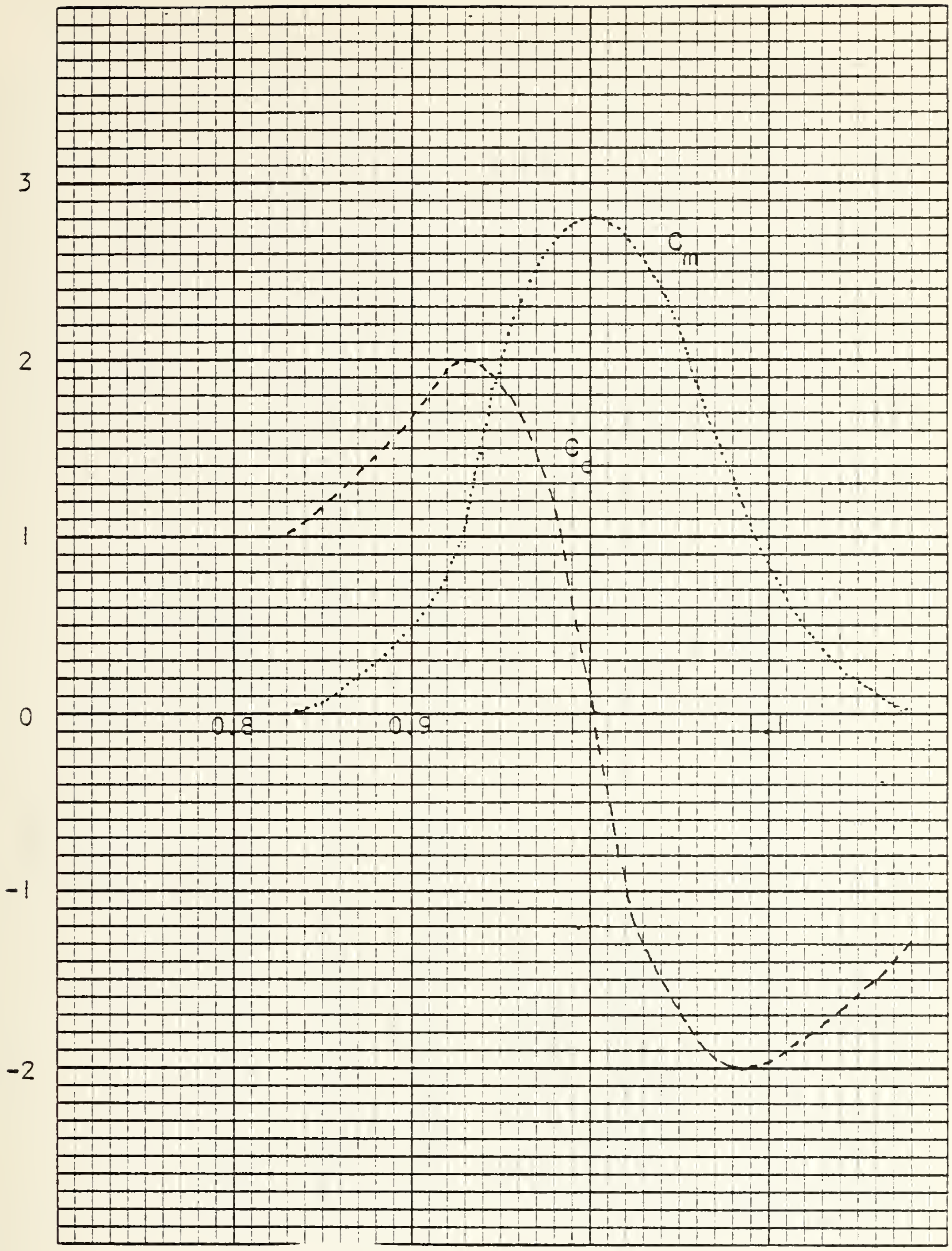

Fig. 15. C_d and C_m versus f_c/f_s for A/D = 0.2, from Hartlen and Currie [3].

APPENDIX A:

COMPUTER PROGRAM

```
C
C
C GIVEN:
C     1. POINTS OF THE CURVE DEFINING CML AS A FUNCTION OF VT/D
C     2. RELATIVE DENSITY = DENS.FLUID/DENS.CYLINDER
C     3. DAMPING RATIO
C     4. VTN/D
C     5. A/D
C     6. POINTS OF THE CURVE DEFINING (CLMX)/(A/D),FOR THE GIVEN A/D,
C        AS A FUNCTION OF VT/D
C
C THIS PROGRAM:
C     1. CALCULATES CAPITAL OMEGA = W/WN
C     2. PLOTS THE SOLUTION OF THE EQUATION OF MOTION
C
C
      EXTERNAL F
      INTEGER*4  ITB(12)/12*0/
      REAL*4  RTB(28)/28*0.0/
      EQUIVALENCE (TITLE,RTB(5))
      REAL*8 TITLE(12)
      DIMENSION XCMW(30),YCMW(30)
      DIMENSION THAU(900),Y(900)
      COMMON XCLMX(30),YCLMX(30)
      COMMON RDEN,Z,VTNOD
      PI=3.14159
C
C READ THE 22 COORDINATES OF THE CML VS. VT/D CURVE
C      THE 23 COORDINATES OF THE (CLMX)/(A/D) VS. VT/D CURVE,AND
C      THE VALUES OF THE REL.DENS.,ZETA,VTN/D,AND A/D
C
      READ(5,11)(XCMW(I),YCMW(I),I=1,22)
      READ(5,11)(XCLMX(I),YCLMX(I),I=1,23)
      DO 50 K=1,10
      READ(5,10,END=1000)RDEN,Z,VTNOD,ACD
C
C FIND THE VALUE OF CAPITAL OMEGA USING SUBROUTINE ZEROIN
C
      B=VTNOD/4.12
      C=VTNOD/7.20
```

```
      CALL ZEROIN(F,B,C,0.005,0.005,IFLAG)
      GO TO (1,1,1,1,2),IFLAG
    2 WRITE(6,15)B,C,IFLAG
      STOP
    1 RESIDL=F(B)
      WRITE(6,15)B,RESIDL,IFLAG
      CAPOM=B
      VETOD=VTNOD/CAPOM
C
C INTERPOLATE THE TWO GIVEN CURVES USING SUBROUTINE RAPINT
C
      CALL RAPINT(VBTOD,23,XCLMX,YCLMX,CLMXDA)
      CALL RAPINT(VBTOD,22,XCMW,YCMW,CMW)
C
C OBTAIN THE SOLUTION OF THE EQUATION OF MOTION
C
      BLIT=((VBTOD**2)*CLMXDA)/(2.0*PI**3)
      CDWMOD=SQRT(BLIT**2-CMW**2)
      CONST=(16./(3.*PI**2))*AOD
      CDW=CDWMOD/CONST
      ALPHA=ATAN(CMWMOD/CMW)
      PHI=ATAN((2.*Z*CAPOM)/(1.-CAPOM**2))
      GAMA=ALPHA+PHI
      AUX4=SQRT(1.-Z**2)
      AUX5=Z-(CAPOM*COTAN(GAMA))
      BETA=ATAN(AUX4/AUX5)
      YH=-SIN(GAMA)/SIN(BETA)
      DO 500 I=1,900
      THAU(I)=(I-1)*0.5
      HOMOG=YH*EXP(-Z*THAU(I))*SIN(AUX4*THAU(I)+BETA)
      PART=-SIN(CAPOM*THAU(I)-GAMA)
      Y(I)=HOMOG+PART
  500 CONTINUE
C
C WRITE THE RESULTS
C
      WRITE(6,16)
      WRITE(6,13)RDEN,Z,VTNOD,AOD
      WRITE(6,12)CAPOM
      WRITE(6,17)(I,THAU(I),Y(I),I=1,900)
C
C PLOT THE RESULTS
C
      ITB(3)=9
      ITB(4)=13
      RTB(1)=50.0
      READ(5,18)TITLE
      CALL DRAWP(900,THAU,Y,ITB,RTB)
```

```
   50 CONTINUE
C
C  FORMAT STATEMENTS FOLLOW
C
   10 FORMAT(4F10.2)
   11 FORMAT(10X,2F10.2)
   12 FORMAT(1H0,10X,'CAPITAL OMEGA =',F10.3//)
   13 FORMAT(1H0//5X,'GIVEN:'//10X,'REL.DENS.=',F6.3,10X,
     1'DAMP.RATIO=',F7.4,10X,'VTN/D=',F6.3,10X,'A/D=',F6.2///
     25X,'WE OBTAIN:'//)
   15 FORMAT(1H1,10X,2F10.3,I10)
   16 FORMAT(1H1,10X,'FINAL RESULTS:'/10X,'**************'////)
   17 FORMAT(5X,I5,10X,F10.4,10X,F10.4)
   18 FORMAT(6A8)
 1000 STOP
      END
      FUNCTION F(CAPOM)
      COMMON XCLMX(30),YCLMX(30)
      COMMON RDEN,Z,VTNCD
      PI=3.14159
      VBTOD=VTNCD/CAPOM
      CALL RAPINT(VBTOD,23,XCLMX,YCLMX,CLMXDA)
      BLIT=((VBTOD**2)*CLMXDA)/(2.0*PI**3)
      E1=RDEN*BLIT
      AUX1=(1.0-CAPOM**2)**2
      AUX2=(2.0*Z*CAPOM)**2
      E2=(SQRT(AUX1+AUX2))/(CAPOM**2)
      F=E1-E2
      RETURN
      END
      SUBROUTINE ZEROIN(F,B,C,ABSERR,RELERR,IFLAG)
      U=9.0E-7
      RE=AMAX1(RELERR,U)
      IC=0
      ACBS=ABS(B-C)
      A=C
      FA=F(A)
      FB=F(B)
      FC=FA
      KOUNT=2
      FX=AMAX1(ABS(FB),ABS(FC))
    1 IF(ABS(FC).GE.ABS(FB))GO TO 2
      A=B
      FA=FB
      B=C
      FB=FC
      C=A
      FC=FA
```

```
    2     CMB=0.5*(C-B)
          ACMB=ABS(CMB)
          TOL=RE*ABS(B)+ABSERR
          IF(ACMB.LE.TOL)GO TO 8
          IF(KOUNT.GE.500)GO TO 12
          P=(B-A)*FB
          Q=FA-FB
          IF(P.GE.0.0)GO TO 3
          P=-P
          Q=-Q
    3     A=B
          FA=FB
          IC=IC+1
          IF(IC.LT.4)GO TO 4
          IF(8.0*ACMB.GE.ACBS)GO TO 6
          IC=0
          ACBS=ACMB
    4     IF(P.GT.ABS(Q)*TOL)GO TO 5
          B=B+SIGN(TOL,CMB)
          GO TO 7
    5     IF(P.GE.CMB*Q)GO TO 6
          B=B+P/Q
          GO TO 7
    6     B=0.5*(C+B)
    7     FB=F(B)
          IF(FB.EQ.0.0)GO TO 9
          KOUNT=KOUNT+1
          IF(SIGN(1.0,FB).NE.SIGN(1.0,FC))GO TO 1
          C=A
          FC=FA
          GO TO 1
    8     IF(SIGN(1.0,FB).EQ.SIGN(1.0,FC))GO TO 11
          IF(ABS(FB).GT.FX)GO TO 10
          IFLAG=1
          RETURN
    9     IFLAG=2
          RETURN
   10     IFLAG=3
          RETURN
   11     IFLAG=4
          RETURN
   12     IFLAG=5
          RETURN
          END
          SUBROUTINE RAPINT(X,N,XN,YN,Y)
C
C
C            INPUT:
```

```
C
C              N= NUMBER OF POINTS WHERE THE COORDINATES OF THE GIVEN FUNCTION
C                 ARE KNOWN
C              XN,YN = COORDINATES OF THE GIVEN FUNCTION
C              X= ABSCISSA OF THE POINT WHOSE ORDINATE WE ARE SEEKING
C
C           OUTPUT:
C              Y= ORDINATE CORRESPONDING TO THE GIVEN VALUE OF X
C
C
      DIMENSION XN(30),YN(30)
      DO 50 ID=1,N
      IF(X.EQ.XN(ID)) GO TO 100
      IF(X.LT.XN(ID))GO TO 200
   50 CONTINUE
      WRITE(6,10)
   10 FORMAT(1H1,'****ERROR: X OUTSIDE LIMITS'///)
      STOP
  100 Y=YN(ID)
      GO TO 500
  200 ID=ID-1
      IDP=ID+1
      AUX=(X-XN(ID))/(XN(IDP)-XN(ID))
      Y=YN(ID)+(YN(IDP)-YN(ID))*AUX
  500 RETURN
      END
```

APPENDIX B:

COMPUTER PROGRAM

```
C
C
C GIVEN:
C     1. POINTS OF THE CURVE DEFINING CML AS A FUNCTION OF VT/D
C     2. RELATIVE DENSITY= DENS.FLUID/DENS. CYLINDER
C     3. DAMPING RATIO
C     4. VTN/D
C     5. A/D
C     6. CAPITAL OMEGA = W/WN
C     7. CLMX FOR THE GIVEN A/D AND VT/D
C
C THIS PROGRAM:
C        PLOTS THE SOLUTION OF THE EQUATION OF MOTION
C
C
      INTEGER*4  ITB(12)/12*0/
      REAL*4  RTB(28)/28*0.0/
      EQUIVALENCE (TITLE,RTB(5))
      REAL*8 TITLE(12)
      DIMENSION XCMW(30),YCMW(30)
      DIMENSION THAU(1200),Y(1200)
      PI=3.14159
C
C READ THE 22 COORDINATES OF THE CML VS. VT/D CURVE, AND
C        THE VALUES OF THE OTHER GIVEN INPUTS MENTIONED BEFORE
C
      READ(5,11)(XCMW(I),YCMW(I),I=1,22)
      DO 50 K=1,10
      READ(5,10,END=1000)RDEN,Z,VTNOD,AOD,CAPOM,CLMX
      VBTOD=VTNOD/CAPOM
      CLMXDA=CLMX/AOD
C
C INTERPOLATE THE GIVEN CURVE USING SUBROUTINE RAPINT
C
      CALL RAPINT(VBTOD,22,XCMW,YCMW,CMW)
C
C OBTAIN THE SOLUTION OF THE EQUATION OF MOTION
C
      BLIT=((VBTOD**2)*CLMXDA)/(2.0*PI**3)
      CCWMOD=SQRT(BLIT**2-CMW**2)
```

```
      CONST=(16./(3.*PI**2))*AOD
      CDW=CDWMOD/CONST
      ALPHA=ATAN(CMWMOD/CMW)
      PHI=ATAN((2.*Z*CAPOM)/(1.-CAPOM**2))
      GAMA=ALPHA+PHI
      AUX4=SQRT(1.-Z**2)
      AUX5=Z-(CAPOM*COTAN(GAMA))
      BETA=ATAN(AUX4/AUX5)
      YH=-SIN(GAMA)/SIN(BETA)
      DO 500 I=1,1200
      THAU(I)=(I-1)*0.5
      HOMOG=YH*EXP(-Z*THAU(I))*SIN(AUX4*THAU(I)+BETA)
      PART=-SIN(CAPOM*THAU(I)-GAMA)
      Y(I)=HOMOG+PART
  500 CONTINUE
C
C WRITE INPUT DATA
C
      WRITE(6,16)
      WRITE(6,13)RDEN,Z,VTNOD,AOD,CAPOM,CLMX
C
C PLOT SOLUTION OF THE EQUATION OF MOTION
C
      ITB(3)=9
      ITB(4)=13
      RTB(1)=100.
      RTB(2)=0.4
      READ(5,18)TITLE
      CALL DRAWP(1200,THAU,Y,ITB,RTB)
   50 CONTINUE
C
C FORMAT STATEMENTS FOLLOW
C
   10 FORMAT(6F10.2)
   11 FORMAT(10X,2F10.2)
   13 FORMAT(1H0//5X,'GIVEN:'//10X,'REL.DENS.=',F6.3,10X,
     1'DAMP.RATIO=',F7.4,10X,'VTN/D=',F6.3,10X,'A/D=',F6.2//
     210X,'CAPITAL OMEGA=',F6.3,10X,'CLMX=',F6.2///
     35X,'WE OBTAIN:'//)
   16 FORMAT(1H1,10X,'FINAL RESULTS:'/10X,'**************'////)
   17 FORMAT(5X,I5,10X,F10.4,10X,F10.4)
   18 FORMAT(6A8)
 1000 STOP
      END
      SUBROUTINE RAPINT(X,N,XN,YN,Y)
C
C
C          INPUT:
```

```
C             N= NUMBER OF POINTS WHERE THE COORDINATES OF THE GIVEN FUNCTION
C                ARE KNOWN
C             XN,YN = COORDINATES OF THE GIVEN FUNCTION
C             X= ABSCISSA OF THE POINT WHOSE ORDINATE WE ARE SEEKING
C
C          OUTPUT:
C             Y= ORDINATE CORRESPONDING TO THE GIVEN VALUE OF X
C
C
      DIMENSION XN(30),YN(30)
      DO 50 ID=1,N
      IF(X.EQ.XN(ID)) GO TO 100
      IF(X.LT.XN(ID))GO TO 200
   50 CONTINUE
      WRITE(6,10)
   10 FORMAT(1H1,'****ERROR: X OUTSIDE LIMITS'///)
      STOP
  100 Y=YN(ID)
      GO TO 500
  200 ID=ID-1
      IDP=ID+1
      AUX=(X-XN(ID))/(XN(IDP)-XN(ID))
      Y=YN(ID)+(YN(IDP)-YN(ID))*AUX
  500 RETURN
      END
```

LIST OF REFERENCES

1. Parkinson, G. V., "Mathematical Models of Flow-Induced Vibrations on Bluff Bodies", Flow-Induced Structural Vibrations (ed. Eduard Naudascher), Springer-Verlag, Berlin, 1974, pp. 81-127.

2. Currie, I. G., Hartlen, R. T., and Martin, W. M., "The Response of Circular Cylinders to Vortex Shedding", Flow-Induced Structural Vibrations (ed. Eduard Naudascher), Springer, Verlag, Berlin, pp. 128-157.

3. Hartlen, R. T. and Currie, I. G., "Lift-Oscillator Model of Vortex-Induced Vibration", Proc. ASCE, EMS, Oct. 1970, pp. 577-591.

4. Meyers, D. W., Transverse Oscillations of a Circular Cylinder in Uniform Flow, Master of Science Thesis, Naval Postgraduate School, 1975.

5. Fortik, D., Forced Oscillations of a Cylinder in Uniform Flow, Master of Science Thesis, Naval Postgraduate School, 1976.

6. Morison, J. R., et al., "The Force Exerted by Surface Waves on Piles", Petroleum Transactions, v. 189, 1950, pp. 149-157.

7. Boyce, W. E. and DiPrima, R. C., Elementary Differential Equations and Boundary Value Problems, 2nd ed., John Wiley & Sons, 1969, pp. 110-122.

8. Thomson, W. T., Theory of Vibration with Applications, Prentice Hall, 1972, p. 46.

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R2195 Raposo
c.1 Transverse oscilla-
tions of a cylinder in
uniform flow.